A slope-density formula

If it is decided for administrative or other reasons that the number of dwelling units permitted on a parcel should depend only on the topography of the parcel and not on the manner in which it is subdivided, then the slope-density formula must take a specific form. This is derived as follows:

Let N be the number of houses permitted by the slope-density formula on a parcel of area A and average slope s. If this parcel is divided into two sub-parcels with areas A_1 and A_2 and average slopes s_1 and s_2, the above condition requires that the number of houses (N_1 and N_2) allowed on each must add up to N. That is if

$$A = A_1 + A_2 \qquad (1)$$

we require that

$$N = N_1 + N_2 \qquad (2)$$

Let a(s) be the minimum lot size permitted on a parcel of average slope s. Then

$$N = \frac{A}{a(s)} \quad , \quad N_1 = \frac{A_1}{a(s_1)} \quad , \quad N_2 = \frac{A_2}{a(s_2)}$$

from (2) we have

$$\frac{A}{a(s)} = \frac{A_1}{a(s_1)} + \frac{A_2}{a(s_2)} \qquad (3)$$

and (3) will be satisfied only if a(s) is of the form

$$a(s) = \frac{a_0}{1-bs} \qquad (4)$$

where a_0 and b are constants to be adjusted to satisfy the planning requirements of the community (if possible). It is seen that a_0 represents the minimum permissible lot size for zero slope. The formula can be made to pass through any lot size a* at slope s* by setting

$$b = \frac{a^* - a_0}{a^* s^*}$$

Thus the Portola Valley SD-I formula passes through $a^* = 7$ acres at $s^* = .50$ and hence

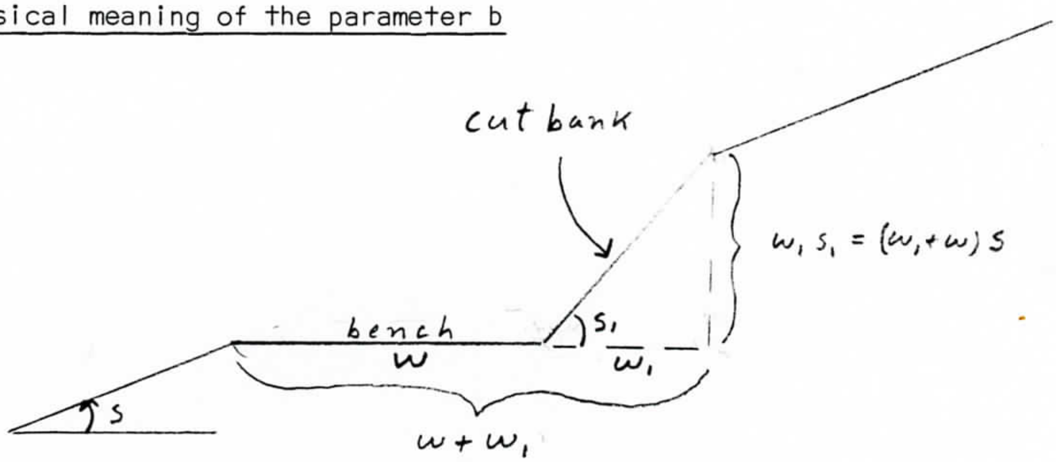
$$b = \frac{7-1}{7 \times .5} = \frac{6}{3.5} = 1.714$$

For SD-I, $a_0 = 1$, hence the formula for SD-I is

$$a = \frac{1}{1-1.714s}$$

where a is the minimum lot size for a parcel of average slope s.

<u>A physical meaning of the parameter b</u>



Consider a bench of width w running parallel to a contour on a hillside with natural slope s. Let the inclination of the cut bank be s_1. Then to create a flat space of width w it is necessary to destroy the natural cover over a width $w + w_1$. The height of the cut bank is given by $(w + w_1)s$ and also by $w_1 s_1$. Hence

$$(w_1 + w)s = w_1 s_1$$

or

$$\frac{w_1}{w} = \frac{s}{s_1 - s}$$

and

$$\frac{w_1 + w}{w} = \frac{1}{1 - \frac{1}{s_1} s} \qquad (5)$$

This says that to create a flat space of any width on a slope, s, it is necessary to destroy the natural cover over a strip $\frac{1}{1-\frac{1}{s_1}s}$ times as wide, where s_1 is the angle of the cut bank. The same relation applies to flat spaces created by fill.

People live, play, park their cars, and (to a large extent) drive them on flat spaces. Assume for the moment that each dwelling unit has roughly the same amount of flat space for such purposes, and that this flat space is achieved by grading. The amount of natural cover destroyed to create this flat space goes up with natural slope as shown in equation (5). Hence if we wish to keep the overall disturbance of the natural cover the same on hill sides as on flat land, the lot size should increase with slope as indicated in equation (5), i.e., the slope formula should be

$$a(s) = \frac{a_0}{1 - \frac{1}{s_1} s} \qquad (6)$$

It is a surprising coincidence that this is the same formula as that suggested earlier on administrative grounds (equation (4)) as long as we choose

$$b = \frac{1}{s_1} \qquad (7)$$

The maximum slope s_1 permitted for cuts by most communities is 66.7% (1.5:1) and the maximum for fill is 50% (2:1). Hence if all of the flat space is created by cutting, with slopes of the maximum allowable inclination, b in equation (7) takes the value 1.5; if the flat space is all created by fill, b = 2.0. At first sight it appears as another surprising coincidence that the value (b = 1.71) chosen for the Portola Valley SD-1 formula is intermediate between these two numbers. The SD-1 formula says that if the flat space is all achieved with grading, using artificial slopes of 1.71:1, then it is assured that the percentage disturbance to the natural cover will be constant irrespective of slope. Perhaps, however, this is not a coincidence; the formal relations described here might have enough physical reality to have influenced empirical planning judgments.

If s_1 were taken as the maximum allowable artificial slope, b = 1.5 and the curve passes through the point a* = 2 1/2 acres s* = 40%. It is seen from equation (4) that such a curve would pass through 1 1/2 acres at 22% and so on. The smaller values of b help take account of the facts that (1) architectural means will often be employed to minimize grading on steeper slopes, and (2) those portions of a parcel that are most intensely graded will usually be somewhat gentler (initially) than the average.

There is a final point worth noting from equation (5). In order to create a flat space by grading it is of course necessary that the artificial slope (s_1) be greater than the natural slope (s). As these two quantities approach one another, creation of a flat space by grading becomes impossible. Thus on a 45% slope, to create a 10-foot wide flat space by fill requires destruction of the natural cover over a width of 100 feet.

Maintaining constant percentage disturbance to the natural cover irrespective of slope, may or may not be a valid planning concept. These remarks are not addressed to that question. Their main purpose is to demonstrate a physical meaning for the parameter b, equation (4), as this equation now forms the basis for slope-density programs in several communities.

Table of average slope (S%) *versus* minimum average lot size (a net acres). Los Altos Hills.

S%	a (net acres)	Y (yield factor)
0	1.000	1.000
10	1.000	1.000
11	1.022	0.979
12	1.045	0.957
13	1.069	0.936
14	1.094	0.914
15	1.120	0.893
16	1.148	0.871
17	1.177	0.850
18	1.207	0.829
19	1.239	0.807
20	1.273	0.786
21	1.308	0.764
22	1.346	0.743
23	1.386	0.721
24	1.429	0.700
25	1.474	0.679
26	1.522	0.657

S%	a (net acres)	Y (yield factor)
27	1.573	0.636
28	1.628	0.614
29	1.687	0.593
30	1.750	0.571
31	1.818	0.550
32	1.892	0.529
33	1.972	0.507
34	2.059	0.486
35	2.154	0.464
36	2.258	0.443
37	2.373	0.421
38	2.500	0.400
39	2.642	0.379
40	2.800	0.357
41	2.979	0.336
42	3.183	0.314
43	3.415	0.293
44	3.685	0.271
45	4.002	0.250

Guidelines (45% - 50%)

S%	a (net acres)	Y (yield factor)
45	4.002	0.250
46	4.376	0.229
47	4.829	0.207
48	5.385	0.186
49	6.090	0.164
50	7.003	0.143
(55	28	.04)
(56.7	∞	0)

$$a = \frac{1}{1 - 0.02143\ (S\% - 10)}$$

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